

## **Developing Metacognitive and Discursive Activities in The Indonesian Mathematics Education**

### **Results of a Feasibility Study**

Christa Kaune, Elmar Cohors-Fresenborg, Edyta Nowinska, Yansen Marpaung, Novi Handayani

#### ***Abstract***

This article reports on the findings of a German-Indonesian feasibility study, which has been conducted to examine whether a more extensive pilot study could be successful. The objective of the pilot study is to enhance the mathematical skills of Indonesian students in the 7th class by increasing the number of students who can really understand the mathematical concepts and methods introduced in class.

In order to achieve this, a learning environment for the introduction of integers was designed and implemented in class. During its implementation a teaching style has been practised which encourages metacognitive and discursive activities in the students.

In this paper the theoretical background for the construction of a comparing test is set out, several exercises are presented as examples and on the basis of student solutions, taken from the test, the effects of the innovative teaching is demonstrated.

**Keywords:** Metacognition, Microworlds, Mental models, Metaphors, Integers

#### ***Abstrak***

Dalam artikel ini kami melaporkan proyek penelitian desain Jerman-Indonesia yang bertujuan untuk meningkatkan kemampuan matematika siswa di sekolah menengah secara signifikan. Sebagai hasil dari studi banding internasional menunjukkan bahwa hubungan antara metakognisi dan pembelajaran adalah lingkungan belajar untuk instruksi dasar di kelas 7 dirancang untuk meningkatkan kegiatan metakognitif dan diskursif pelajar dan guru dengan jelas. Efektivitas pendekatan ini diuji dalam beberapa kali sekolah menengah.

Dalam tulisan ini, latar belakang teoritis untuk pembangunan lingkungan pembelajaran disajikan, beberapa tugas disajikan sebagai contoh dan analisis produksi mahasiswa dari pelajaran proyek.

**Kata kunci:** Metacognition, Microworlds, Mental models, Metaphors, Integers

From October the first in 2009 until the last of December in 2010, mathematics education researchers at today's Universitas Santa Dharma (Yogyakarta) and the University of Osnabrück jointly conducted a feasibility study called "Development of metacogni-

tive and discursive activities in the Indonesian Mathematics education (MeDIM)“. The study's objective was to investigate whether a more extensive pilot study might be promising, in which enhancing the mathematical skills of students in 7<sup>th</sup> class -- attending Indonesian secondary schools -- is planned to achieve by increasing the percentage of students who really comprehend the mathematical concepts and procedures elaborated in class. For this aim a teaching and learning culture should be established in the test classes, featuring two central aspects:

- Priority to the development of sustainable mental models (how to deal with integers and algebraic transformations) prior over the mediation of factual knowledge and the practice of calculation techniques.
- Increasing metacognitive and discursive activities in class.

The feasibility study aimed to assess the pilot study's design both for the implementation and for the evaluation. In the design for the evaluation the test class plays the part of the future project classes. In the following the focus will be on the evaluation design and the results of its analysis. “Results of the evaluation” means the effects of the designed measures (teaching content and methods) on the teaching-learning process and its outcomes. In this paper we concentrate more on outcomes (students' mathematical skills). An in-depth analysis of teaching-learning situations is intended for further publications.

### ***Realisation and Results of The Feasibility Study***

The educational conception, the theoretical background for the construction of a learning environment (cp. Kaune & Cohors-Fresenborg, 2011), and some tasks and students' productions selected as examples from the project lessons are covered in (Kaune, Cohors-Fresenborg & Nowinska, in this volume). Thereby also the design of the planned pilot study is documented.

The learning environment has been tested at the beginning of the school year 2010/2011 in a grade - seven class at the Sekolah Menengah Pertama in Yogyakarta by a teacher who had been trained for this in Germany. This opportunity was used in the feasibility study to examine the implementation design and to identify variables which influence and determine the success of the implementation.

After completing the implementation in class (26 double lessons, each lasting 90 minutes) an evaluating test was written in August 2010, in this experimental class and an-

other year seven class at the same school (control class). On the basis of the test result students' mathematical achievement had been compared and therefore the impact of the study had been investigated.

We ensured that at the time of the test the same number of mathematics lessons had been taught in both classes. This implies that the students in the test class received a large number of content and training exercises which at first sight are not covered in the Indonesian curriculum. On the other hand they had considerably less time to practice calculation techniques.

### **Construction of The Test Tasks**

The evaluating test aims to assess the students' abilities to calculate with integers in various ways. Furthermore it is of great interest whether and to what extend the students use the models offered in the textbook when calculating. In addition the test assessed their competence in key mathematical operations, which are considered important beyond the topic "operating on integers". Those are the ability to insert number's names and terms for variables in calculation rules, obeying syntactic rules when writing mathematical expressions and the students' comprehension of the meaning of the taught calculation rules.

The tasks had to be constructed in such a way that also the students from the control class would be able to solve them. Therefore any reference to specific features of the learning environment (the contract to operate on integers, in particular instructions to calculate thoroughly in compliance with the contract or to prove a new, unknown mathematical theorem) had to be avoided.

The designed test involved five tasks each requiring different abilities. The test had to be solved in one lesson (45 minutes).

The first task consisted of four technical exercises demanding accurate calculation, i.e. it was a matter of the correct result.

1. Calculate.

- |                             |                              |
|-----------------------------|------------------------------|
| a. $((-24) + (-34)) =$      | b. $((4 + (-9)) + 16) =$     |
| c. $(((-25) + 175) + 25) =$ | d. $((123 + (-23)) - 100) =$ |

The second task tested whether the student is able to perform the process of inserting numbers for variables.

2. You know the commutative law:  $(a + b) = (b + a)$

- a. Please insert for  $a = (-25)$  and for  $b = 15$ .

b. Please insert for  $a = 16$  and for  $b = (-30)$ .

Through the third and fourth exercises two aspects should be assessed: the reasoning skills and the robustness of mental models for integer operations, when the following mathematical facts should be explained:

3. Please write down an explanation why  $(0 + a) = a$ .
4. A student forgot the additive inverse of  $(-17)$ . He wonders whether it is  $17$  or  $(-(-17))$ .

What do you think?

Part a of the fifth task aims to assess the formalisation technique, part b, however, the application of a mathematical rule..

5. There exists a relation between subtraction and addition. If we subtract from a number a second number, we could also add to the first number the additive inverse of the second number. Write down this knowledge with the use of variables.

Please complete:  $( (-30) - 15 ) = \dots$

## Hypotheses and Results

28 students sat the test in the test class, and 29 students in the control class. Due to small number of classes involved and students tested we focus rather on the interpretation of the student solutions in order to illustrate what has been achieved or is achievable and not on detailed statistical analysis. For some of the hypotheses formulated in the following statistical tools are going to be applied in the future pilot study, in order to evaluate the statistical significance of quantitative statements.

### ***Hypothesis 1: Students of the test classes calculate more successful with integers than students of the control class***

To test this hypothesis we examine the solutions to the first task. At first we checked whether the groups differ with respect to the number of correct answers. To this end each correct result was rated with one point, a wrong result with 0 points. So a student could achieve maximally four points in task one.

Therefore the 29 students of the test class could maximally reach 29 times 4, i.e. 116 points. The number of points obtained was 96 (out of 116), i.e. 82,76% of the maximal number of points. The 28 students of the control class could have reached a maximal number of 28 times four, i.e. 122 points. They reached 77 out of the 122

points, which is 68,75% of the maximal number of points. For each subtask the students in the experimental class achieved a higher average then the students in the control class.

In the future pilot study one would expect analogue performance differences, which should prove to be significant.

It should be the aim of mathematics education that students not only calculate correctly but are also able to explain how they derived their result of the calculation. In the design of the teaching -learning environment in the MeDIM project, providing training in explaining and reasoning was considered a high priority. Although the way the test is constructed it does not ask for any explicit explanations of the calculations, many students still offered an additional justification for the calculation steps they performed. Here two types have to be distinguished: On the one hand simple, remembered rules had been cited, on the other hand references to basic mental models or microworlds constituted the justification. First we examine the former case.

***Hypotheses 2: Without being asked for, students of the test class justify their calculations by citing remembered rules more often than students of the control class.***

To verify this hypothesis we as well consult the solutions to the first task. .

In the control class in four out of the 112 subtasks (4%) a justification for the chosen calculation has been indicated.

$$\begin{aligned}
 \text{a. } ((-24) + (-34)) &= 24 + 34 = (-58) \\
 &= -58 \\
 (-) &\text{ dihilangkan tahulu}
 \end{aligned}$$

**Fig. 1: A reference to a rule used in the control class: “First we leave out (-)”**

In the experimental class a justification was provided in seven out of the 116 subtasks (6 %). In all of the cases short notations of rules were written down, which never had been subject in the project lessons. Figure 2 shows a correct solution to the subtask 1a with the student's own abbreviation of the rule “The sum of two negative numbers is a negative number”.

<p>a. <math>(-24) + (-34) =</math></p> $  \begin{array}{r}  -34 \\  -24 \\  \hline  -58  \end{array} +  $ <p><math>(\text{negatif}) + (\text{negatif}) = \text{negatif}</math></p>
--

**Fig. 2: Reference to a used rule in the experimental class**

We explain this as follows: The use of simple rules to be remembered is a typical element in mathematics education. In the project lessons the teacher did not offer such simple rules. The rules found in the test therefore indicate that the students constructed such mnemonic rules themselves. The mathematical quality of the cited rules and their representation differs in both classes.

The fact that students in the project class provide a justification or a comment for their calculation is a sign that the way of teaching changed. The students should have been strongly encouraged to become active metacognitively. This includes the control of one's own thought processes and the reflection upon them.

In the future pilot study one would expect analogue differences, which prove to be significant.

As outlined in Kaune, Cohors-Fresenborg & Nowinska (2011, in this volume), while designing the teaching-learning environment in the MeDIM project, special emphasis was placed on supporting the development of mental models on how to operate on integers and deal with algebraic transformations by the means of providing appropriate microworlds. The mental models should become a tool for the learners to deal with mathematical matters in a sensible way.

First the students' experiences with debt and credit and their intuitive knowledge how to deal with debts had been extended into the metaphor system "Contractual arithmetic". The metaphor system forms the core of the first microworld, in which the learner has the opportunity to organise mathematical facts. This microworld was further expanded into a second microworld by covering the process of reconstructing the existing intuitive knowledge of the learner in class. As a result the experiences were normatively fixed in a contract (an axiomatic system).

A third microworld is constituted by a board game in which movements on the number line are executed with game pieces.

In the control class no microworlds are provided, only examples for the occurrence of negative numbers are given, taken from the alleged everyday life of the students.

While they serve to motivate the introduction of integers and their notation, they do not allow the development of robust basic mental models which help to orientate oneself in mathematics.

***Hypothesis 3: In their calculations students in the control class almost never refer to the examples provided in the introductory sections of the textbook or the introductory lessons, students in the test class, to the contrary, frequently use the offered microworlds to organise mathematical facts in their minds.***

Again we will use the solutions to the first task in order to examine the third hypothesis. While formulating the task special attention had been paid to ensure that no linguistic references to one of the microworlds was provided or suggested.

One cannot expect any references to microworlds in the control class. But also none of the basic models introduced in the textbook or in class (temperature, scales) has been mentioned even once in a solution to a subtask. This shows, that the students did not consider the offered examples helpful.

In the test class the situation is different:

In a total of 22 (out of 116) subtasks (19%) the test students added comments which indicate the utilisation of the offered microworlds. Whether and how many other students utilised a microworld cannot be estimated. To answer this question a different survey method would be necessary in future investigations: an interview or a questionnaire.

### **Analysis of student solutions referring to a microworld**

In the student solutions one can find references to the three microworlds “Crediting and debiting”, “Contract” and “Jumping on the number line”.

In the test of the experimental class, students justified their calculations nine times by referring to the microworld “Crediting and debiting”. The following is a first example of a correct calculation of the term in part a, including a semi-formal notation of a justification for the result in the microworld “Crediting and debiting”:

$((-24) + (-34)) = -58$ krn utang 24 + utang 34 = utang 58
---

**Fig. 3: Panta justifies the result in the microworld “Crediting and Debiting”:**

**“Because debt 24 + debt 34 = debt 58”**

Bella's solution shows as well that while calculating she utilised the microworld "Crediting and debiting" to organise the mathematical facts. Her calculation is syntactically not correct. Presumably she wanted to express: "Calculate everything with debts": " $24+34 = 58$ ".

Hitunglah.

$$((-24) + (-34)) = -58$$

$$\text{utang } 24 + 34 = 58$$

**Fig. 4: A reference to the model world "Crediting and debiting": "Debt  $24 + 34 = 58$ "**

Dian dedicates a lot of writing work and time to justify his results in subtask c by describing a transaction story in the microworld "Crediting and Debiting" that fits the term:

d.  $((123 + (-23)) - 100) = 0$

jadi dia pertama nya punya uang Rp 123 lalu dia -  
lalu setoran debit/hutang sebanyak Rp 23 jadi  $= (123 + (-23)) = 10$

lalu setelah itu ia mengambil uang nya sebanyak  
Rp 100 jadi ia pertamanya habis membayar utang sebesar uang nya  
tgk Rp 100 . lalu ia mengambil uang sebanyak Rp 100 jadi  
uang nya Rp 0 det....

**Fig. 5: Justification for the result in the microworld "Crediting and debiting": "First he has got Rp 123, then he withdraws Rp 23, therefore  $= (123 + (-23))=10$ . After that he withdraws Rp 100. So, first he pays his debt and he has Rp 100 left and he withdraws Rp 100 now his money is Rp 0 ..."**

In the following the student's comment is analysed with respect to the importance of the reference to the amount of money for his solution of a mathematical problem, which had been set by a purely technical formulation.

Referring to the monetary amounts and the transactions the student cannot simply recall an application problem, as each amount of money, e.g 123 Rp, 23 Rp and 100 Rp, is not used in the Indonesian everyday life. The world of transaction became a metaphor, which shapes the way he thinks and even systematizes problems which have no direct equivalent in reality.

Interestingly, the student tries to convey a consistent justification. Having already explained the whole term in the world of transactions, (“At first he owns Rp 123, then he credits his account with a debt of Rp 23, hence  $= (123 + (-23))$ ). After that he makes a withdrawal of Rp 100.”), he jumps back again and comments the intermediate step in the world of transactions (“So, after he has paid his debts, he has Rp 100 left.”) He continues his argumentation from this point (“Now he withdraws Rp 100. Thus he has got Rp 0...”)

In this student's solution relating Mathematics to the reality does not concern coping with the reality with the help of Mathematics, in contrast, the experience in dealing with the reality is used as a metaphor in order to understand the now unfamiliar abstract Mathematics. This is further explained in the previous article (Kaune, Cohors-Fresenborg & Nowinska, 2011, in this volume).

The **microworld** which students in the test class rely on for calculations most frequently is the **”Contract to operate on integers”**. One can find twelve references thereto, as for example in Figure 6:

$$\begin{array}{l}
 a=4 \\
 b=(-9) \\
 c=16
 \end{array}
 \quad
 \begin{array}{l}
 \text{b. } ((a+b)+c)+16 = ((a+b)+c)+16 \quad A+ \\
 = || * 
 \end{array}$$

**Fig. 6: Justification of the result by calculations according to the contract**

The student Andre recalls the contract in his mind, first the associative law. The whole notation shows that Andre really recalls the paragraph  $A^+$  (associative law of the addition) in the way it has been agreed upon in class: First there is the notation of  $a, b, c$  under the term. In order to apply the paragraph to the equation, one has to substitute the terms. It has been agreed in class to cover this in paragraph  $*$ , accordingly Andre cites this paragraph on the right margin. On the left margin he denotes the necessary term substitutions. In conclusion, his notation shows that Andre really orients himself in the microworld “Contract” rather than just writing down reasons for a calculation step.

By applying the associative law Andre is able to determine the positive result of  $((-9) + 16)$  in an easier way than the negative result of the given sub-term  $(4 + (-9))$ .

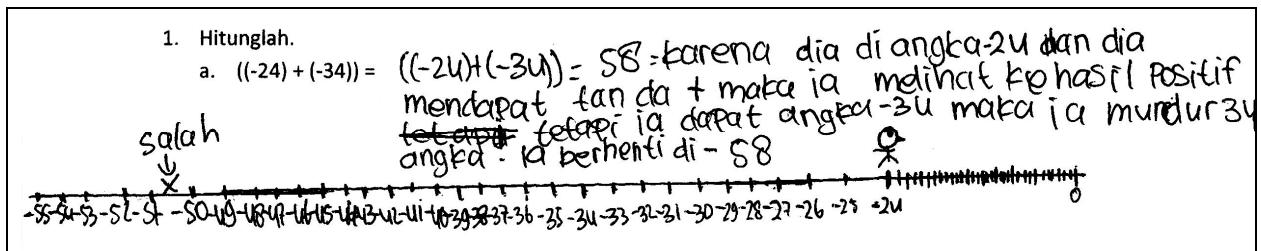
In the 122 solutions to the subtask 1b, handed in by students of the control class, one can find only one reference to an arithmetic law that has been utilised.

$$\begin{aligned}
 b. ((4 + (-9)) + 16) &= (4 - 9) + 16 \\
 &= (-5) + 16 \\
 &= 16 + (-5) \rightarrow \text{dibalik} \\
 &= \underline{\underline{11}}
 \end{aligned}$$

**Fig. 7: A comment to the use of the commutative law “exchange”**

This description of the use of the commutative law is not very elaborate. Here mentioning the law does not represent the use of a microworld, in which the student understands the status of calculation laws/axioms/laws of arithmetic, but it is a local justification of the calculation step through a memorized rule.

The microworld “jumping on the number line” is used as a help only once:



**Fig. 8: Addition of two integers, justification of the result in the microworld**

**“jumping on the number line”:**

**Because he was at number -24 and he gets a +, so he looks to the positive direction, but he gets -34 so he walks backwards 34 numbers. He stops at -58.”**

The student Dodi correctly translates the first summand into the position of the playing piece: “He was at -24”, the function sign for addition into the position of the function sign dice “he gets a + on the sign dice”. This is correctly transferred into a turn of the piece “hence he looks in the positive direction.” The second summand is again transferred correctly into a position on the number dice “Then he gets a -34 on the number dice” and this then into the piece's movement: “Thus he moves 34 steps backwards.” The final position of the piece is correctly determined “He stops at -58”. His illustrations show the starting position of the movement and the playing piece. Then he determines in a way, which cannot be reconstructed, that he has to place the cross onto the -52. He realises his mistake, and marks his cross, placed at the -52, as wrong – “salah”. Hence he must have monitored his activities at this point.

In the algebraic representation (in the first line), to the contrary, he did not monitor his activities: As a result of the calculation he writes down 58, despite the fact that 58 occurs neither in his drawing nor in his verbalisation of the game.

So in order to correctly perform the calculation required by the task, he does not lack an underlying concept, but he has to monitor the transfer of the in the microworld correctly determined result into the algebraic representation.

The fact that only a few students chose the game as a basic mental model for the calculations might be due to the large numbers and the with this associated, big effort to draw the numberline. During the lessons this game proved to be very useful for the development of a second world of mental models. For the technical application, however, the microworld “Contract” is obviously easier to handle especially because of its formal compactness.

***Hypothesis 4: Students who at least once in the technical tasks (tasks 1a – 1d) show through a comment that they recourse to a microworld while calculating calculate in a more reliable way.***

Seven out of 67 students showed through a comment, that they use a microworld when calculating. 25 out of the 28 answered subtasks are solved correctly (89%). In two other subtasks the mistake was made while copying the result, which had been solved correctly in the microworld, into the algebraic representation (cp. Fig. 8). With respect to the evaluation, they are still considered “wrong”. The remaining 60 students solved 154 out of 240 subtasks correctly (60%).

Thus the hypothesis has been confirmed. Furthermore this result is even statistically significant despite the small number of subjects ( $t(17,16)=2,36$ ,  $p<0,05$ ). This shows that it is worthwhile to pursue this question in a pilot study in greater detail.

We suppose that the better performance can be explained by the fact that the spontaneous recourse to suitable microworlds can be considered an indication of underlying monitoring and reflection processes, which are known in the literature to increase the frequency of correct solutions.

While the hypotheses so far were more concerned with the technical abilities, the next hypotheses will focus more on the mathematical reasoning of the students.

**Hypothesis 5: Students in the test class are better at arguing mathematically than students in the control class.**

To address this hypothesis we analysed the solutions to the third task. The task was to explain the meaning of the law  $(0 + a) = a$ . The students in the experimental class show, with a mean of 2,88 out of 4 points, a significantly better performance than the students in the control class, reaching 1,23 points on average, ( $t(55) = 3.848, p < 0.05$ ). This demonstrates that the microworlds offered in the experimental class have been incorporated by the students and improve their ability to argue mathematically:

karena 0 itu sama saja orang baru membuka rekening, sedangkan a itu adalah pertama kali ia menyetor dan menjadi saldo baru

**Fig. 9: Hosye's Explanation of the law of arithmetic in the microworld “Credit-ing and debiting”:**

**Because 0 is the same as when someone opens an account, while “a” is the first amount that he deposits and it becomes the new balance.”**

Reading the equation Hosye interprets it by reference to the microworld “Credit-ing and debiting”. For her the equation describes the first transaction after opening a bank account.

In contrast to Holy's, in Nina's explanation one cannot find a reference to the microworld “Credit-ing and Debiting”, but to the microworld “Contract”.

Itu sudah ada pasalnya ( $N^+$ )  
Jadi pasalnya :  $N^+ \cdot (0 + a) = a$ .  
Atau jika kita mulai dari 0 dan ditambah dengan a.  
Contoh:  $0 + 12 = 12$ .  
Jadi, jika 0 ditambah dg. bilangan lain, hasilnya bilangan itu sendiri;

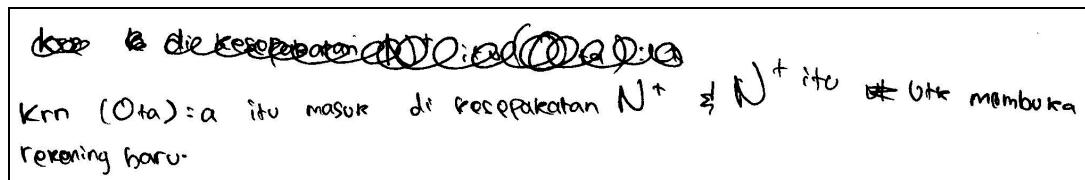
**Fig. 10: Nina points out the similarities to a law:**

**It is the theorem ( $N^+$ ). So the theorem is:  $N^+ = (0 + a) = a$**

$N^+$ , the name for the paragraph agreed on in class, originates from this microworld.

15 of 29 students of the test class refer to this paragraph  $N^+$ . Thus, the microworld “Contract” is most often mentioned in connection with the explanation of the calculation rule.

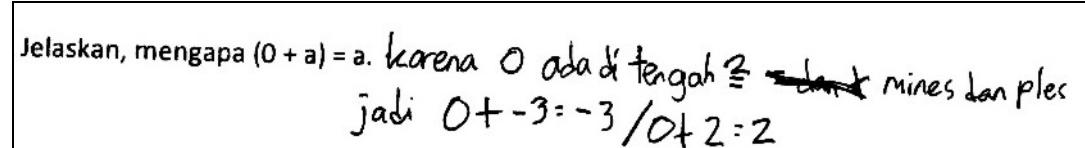
Panda's solution reflects that the microworld "Contract" is not independent of the microworld "Crediting and Debiting", but has been developed on the basis of the later. Since she refers to the fact that contracts serve to regulate behaviour, one can see that she really understands what is meant by calculating in compliance to a contract.



**Fig. 11:** Panta points out the similarity to a contract, and also the use of the paragraph agreed upon in the microworld "Crediting and Debiting": "**Because  $(0+a) = a$  is in the agreement  $N^+$  and  $N^+$  is for opening a new account.**"

Explanation possibilities, like the three solutions above, rely upon the microworlds, introduced in the project lessons by the learning environment. These are not accessible for students of the control class. In order to explain the rule, they could only use the introductory examples in the text book (the use of scales), a colloquial description of the nature of zero to be the neutral element of addition, or a direct reference to the rule, depicted in a box inside the textbook in use (Adinawan et al., 2006, S.10).

6 of the 28 students in the control class did not solve the task in an appropriate way, they probably did not understand it.



**Fig. 12:** Example of an inconsistent explanation provided by a student in the control class:

**"Because 0 is in the middle of minus and plus. Thus  $0 + -3 = -3$  /  $0 + 2 = 2$ "**

The first part of Hugo's statement "Since 0 is between minus and plus." probably refers to the position of zero on the number line between the positive and negative numbers. The examples following, however, do not exhibit any relation to the former statement.

Twelve students only verbalised the equation, but did not explain it. Two students provided an example for the formalised rule.

Only eight students provided an answer appropriate for the task, two of them Sebatinus:

Jelaskan, mengapa  $(0 + a) = a$ .  
 karena bilangan 0 merupakan bilangan identitas  
 yg jika ditambahkan / dikurangi hasilnya tetap  
 angka yg ditambahkan

**Fig. 13: „Because 0 is an identity element.**

**If a number is added or subtracted to this number, the result is the number,  
 which is added.”**

and Augustina:

Jelaskan, mengapa  $(0 + a) = a$ .  
 Karena, jika bilangan ~~perjumlahan~~, apa saja  
 dijumlahkan dengan nol (0) maka hasilnya  
 tetap bilangan itu sendiri.

**Fig. 14: Augustina expresses the statement colloquially:**

**“Because, if any number is added to zero (0), the result is the number.“**

We interpret Sebatinus and Augustinas answers as adequate verbalisations of the law depicted in a box inside the textbook in use (Adinawan et al. 2006, S. 10).

In his/her explanation, none of the students in the control class uses the introductory examples from the textbook on how to deal with integers (scales). They do not help the students to explain arithmetic laws. Also in the textbooks we analysed, the introductory examples are used to explain the existence of whole numbers, but not to clarify calculation rules.

We would like to draw attention to the fact that considerably more students in the test class than in the control class (15 out of 29 versus 8 out of 28) are able to refer in a meaningful way to a paragraph or a law of arithmetic while reasoning mathematically. In addition, five students in the test class refer to the microworld “Crediting and Debiting” during their argumentation.

All in all, the analysis of hypothesis 5 shows the huge potential of the innovation with respect to mathematical reasoning.

The efforts to change the teaching culture should be reflected in the answers of the students. A central endeavour of the teaching staff in the project lessons has been to

advise the students to provide a reason for their statements, often also to put it in writing. It was the purpose of exercise 4 to check whether this training bears a positive effect.

***Hypothesis 6: Students in the test class are more capable of complex mathematical reasoning than students in the control class***

In order to answer the task

*A student forgot the additive inverse of (-17). He wonders whether it is 17 or (-(-17)).*

*What do you think?*

One had to argue in the following complex way: According to the procedure in the test class lessons, one obtains the inverse of a number by writing the minus sign in front of it and then putting the term in brackets, i.e. by changing  $(-17)$  into  $(-(-17))$ . On the other hand the theorem  $(-(-a)) = a$  has been proven in class. Consequently one knows that  $(-(-17)) = 17$ . Therefore the directly constructed inverse to  $(-17)$  is the number  $(-(-17))$ , which is known to be equal to 17.

Similarly, a student of the control class could have argued on the basis of the calculation laws introduced in the textbook.

It is necessary for the argument that the student names 17 as well as  $(-(-17))$  as an inverse to  $(-17)$ . 9 of the 29 students in the test class and 2 of 28 students in the control class did this. The classes noticeably differ as well in providing or not providing a reason for the inverse stated. Not a single student mentions the subtle difference between the statements used in the argumentation, a definition on the one hand and a proven theorem on the other hand. One should notice that in both classes the students often do not respond in an adequate way to the logical structure of a twofold question, as they start their response with “Yes, this is correct.” We regard this as an indication that in the Indonesian school education the questions posed are too often questions to be answered with yes or no respectively.

The analysis of task 4 shows that it is worthwhile to include such exercises in the pilot study.

### ***Outlook***

As result of the feasibility it can be stated that a broad scale pilot study with the planned design is very promising. In particular possibilities have been revealed to considerably increase the students' understanding for mathematical concept formation

and their reasoning competence. This is the reason why we applied for financial support for such a pilot study to the catholic charity organisation MISEREOR in Germany. Since several schools in Central Java had expressed their interest in participating, MISEREOR approved the application by now. The pilot study began on the first of April 2011.

### ***References***

- Adinawan, M. Cholik et. al. (2006). *Mathematika untuk SMP KELAS VII Semester 1A*. Jakarta: Erlanger.
- Kaune, C. & Cohors-Fresenborg, E. (2011). *Perjanjian untuk Berhitung. Buku Pegan-gan bagi Guru*. Osnabrück: Forschungsinstitut für Mathematikdidaktik.
- Kaune, C., Cohors-Fresenborg, E. & Nowinska, E. (2011). Development of metacognitive and discursive activities in Indonesian Maths Teaching. A theory based design and test of a learning environment. (in print)
- Marsigit (2008). *Mathematics for Junior High Scholl Year VII*. Jakarta: Yudhistira.

### **Christa Kaune**

Institut für Kognitive Mathematik, Universität Osnabrück, Germany  
Email: ckaune@uos.de

### **Elmar Cohors-Fresenborg**

Institut für Kognitive Mathematik, Universität Osnabrück, Germany

### **Edyta Nowinska**

Institute MATHEESIS, Pyzdry, Poland

### **Yansen Marpaung**

Universitas Sanata Dharma, Yogyakarta

### **Novi Handayani**

Universitas Sanata Dharma, Yogyakarta